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LETTER TO THE EDITOR

Universality of critical existence probability for percolation on three-dimensional lattices

Chai-Yu Lin[†], Chin-Kun Hu^{‡§} and Jau-Ann Chen[‡][†] Institute of Physics, National Tsing Hua University, Hsinchu 30043, Taiwan[‡] Institute of Physics, Academia Sinica, Nankang, Taipei 11529, Taiwan

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Abstract. Using a histogram Monte Carlo simulation method, we calculate the existence probability E_p for bond percolation on simple cubic (sc) and body-centred cubic (bcc) lattices, and site percolation on sc lattices with free boundary conditions. The R_1 spanning rule considered by Reynolds, Stanley, and Klein is used to define percolating clusters. We find that E_p for such systems has very good finite-size scaling behaviour and the value of E_p at the critical point is universal and is about 0.265 ± 0.005 .

The existence probability $E_p(L, p)$, which is the probability that a system of linear size L percolates at occupation probability p , is an essential quantity in renormalization group (RG) approaches to percolation problems [1–3]. E_p was called the crossing probability by Langlands *et al* [4, 5] and the spanning probability by Ziff [6]. In the limit $L \rightarrow \infty$, $E_p(L, p)$ approaches the step function $\Theta(p - p_c)$, where p_c is the critical threshold [7]. In this case, if we write $E_p \sim (p - p_c)^a$, then the critical exponent a is zero. Therefore, E_p is also an ideal quantity for studying finite-size scaling functions and universality because we need not scale E_p to plot it as a function of the scaling variable [7–11]. There have already been extensive studies of E_p for percolation on two-dimensional (2d) lattices [2–6, 8–12]. Using a histogram Monte Carlo simulation method (HMCSM) [2, 3], in this letter we study finite-size scaling and universality of E_p for percolation on 3d lattices.

In 1992, Langlands, Pichet, Pouliot, and Saint-Aubin (LPPS) [4] proposed that when aspect ratios for square (sq), honeycomb (hc), and plane triangular (pt) lattices are in the proportions $1:\sqrt{3}:\sqrt{3}/2$, then site and bond percolation on such lattices have the same value of E_p at the critical point. In 1992, Cardy used a conformal theory to write down a formula for critical E_p as a function of aspect ratio for percolation on lattices with free boundary conditions [5], which was confirmed by LPPS's numerical calculations [4]. Cardy and LPPS did not discuss the values of E_p for $p \neq p_c$. In 1995–1996, we applied the HMCSM to calculate E_p , the percolation probability P , and the probability of finding exactly n percolating clusters, W_n , for site and bond percolation on finite sq, hc, and pt lattices [11–14] using the scaling variable $z = (p - p_c)L^{1/\nu}$. By using the LPPS relative proportions of aspect ratios [4] and non-universal metric factors [15], we found universal finite-size scaling functions for E_p , P , and W_n of six different site and bond percolation models on planar lattices [11–14].

§ E-mail address: huck@phys.sinica.edu.tw

In order to investigate the universality of critical E_p in higher dimensions, Stauffer *et al* [16, 17] performed a series of simulations for bond and site-bond percolation on three- to six-dimensional hypercubic lattices. In particular, they found that for both bond and site-bond percolation on cubic lattices with free boundary conditions $E_p(L, p_c)$ approaches 0.42 for L up to 1001 using the R_1 percolation rule defined by Reynolds *et al* [1]. Since Stauffer *et al* did not consider the universality of critical E_p for different types of lattices and their data are not very precise, more work on the universality of critical E_p for $d \geq 3$ is needed.

In this letter, we use the HMCSM [2] to calculate E_p for bond percolation on simple cubic (sc) and body-centred cubic (bcc) lattices, and site percolation on sc lattices, which have free boundary conditions. The bonds (sites) of the first plane are randomly occupied with probability p as are other planes of the lattices. The R_1 spanning rule considered by Reynolds *et al* [1] is used to define percolating clusters. We find that E_p for such systems has very good finite-size scaling behaviour and the value of E_p at the critical point is universal and is 0.265 ± 0.005 , which is different from the value 0.42 obtained by Stauffer and his coworkers [16]. To check the reliability of our result, we prepared two independent computer programs. The results obtained from the two programs are consistent.

In this letter, both sc and bcc lattices are considered to be a cubic system, which is described by an $L \times L \times L$ lattice. Therefore, a $2 \times 2 \times 2$ bcc lattice includes eight sites at the corner and one site in the centre. Here we define the effective linear dimension of the lattice, L_{eff} , as the cubic root of the total number of lattice sites. For example, L_{eff} is equal to 2 and $9^{1/3}$ for the $2 \times 2 \times 2$ sc lattice and $2 \times 2 \times 2$ bcc lattice, respectively.

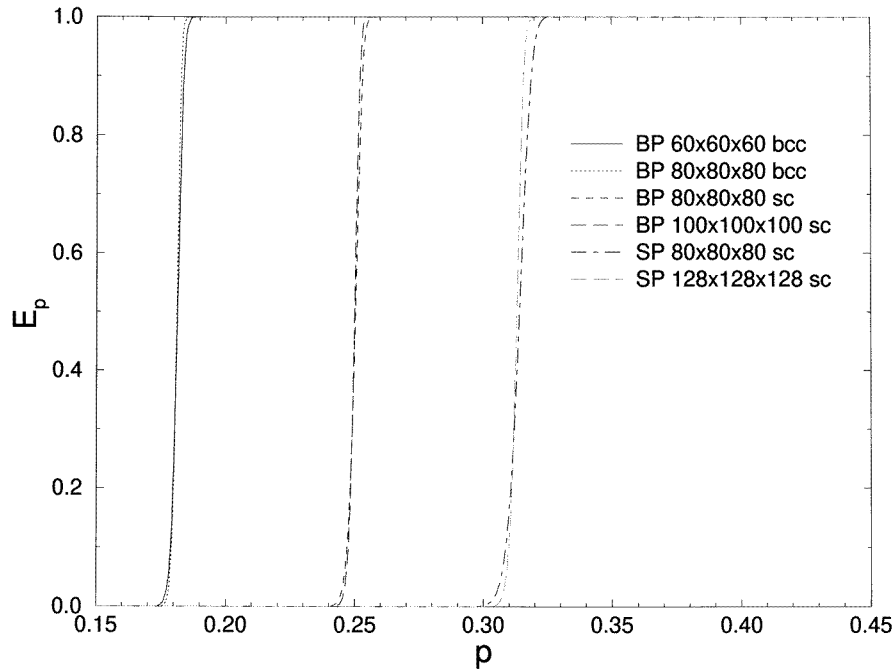


Figure 1. The existence probability $E_p(L, p)$ for bond percolation (BP) on bcc lattices, BP on sc lattices, and site percolation (SP) on sc lattices.

Here we briefly describe the HMCSM for bond percolation on $L \times L \times L$ sc and bcc lattices. The extension to site percolation is straightforward. In bond percolation on a

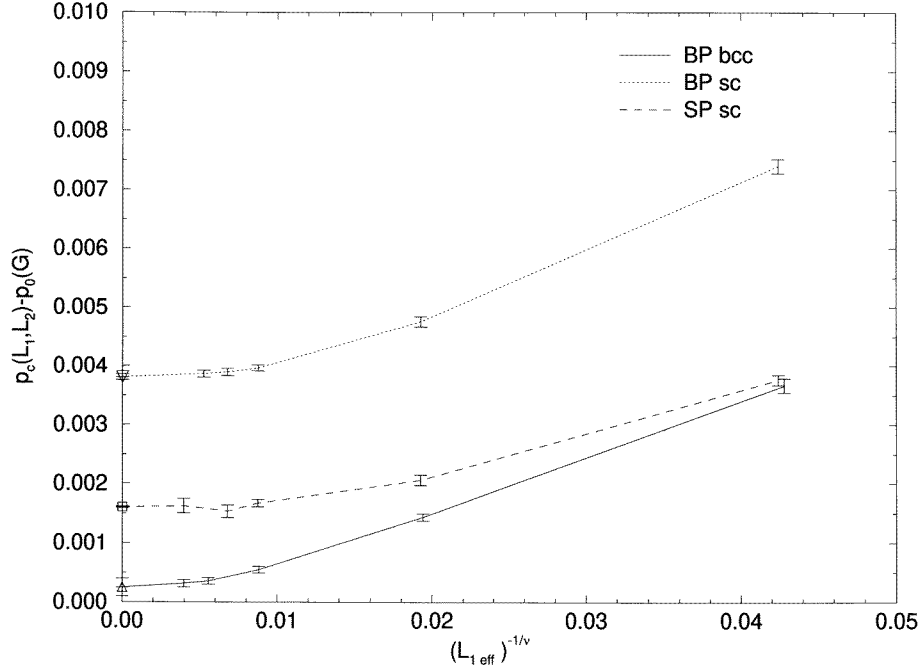


Figure 2. The calculated $p_c(L_1, L_2) - p_0(G)$ as a function of $(L_1^{\text{eff}})^{-1/\nu}$, where $p_c(L_1, L_2)$ is obtained from equation (4), L_{eff} is the cubic root of the number of lattice sites, and values of $p_0(G)$ for BP on bcc lattice, BP on sc lattice, and SP on sc lattice are 0.180, 0.245, and 0.310, respectively. The symbols Δ , ∇ and \square on the vertical axis correspond to 0.18025 ± 0.00015 , 0.248810 ± 0.000050 , and 0.311605 ± 0.000010 , respectively.

d -dimensional lattice G of N sites, E bonds, and linear dimension L , each bond of G is occupied with probability p , where $0 \leq p \leq 1$. A cluster which extends from a given side of G to the opposite side is a percolating cluster. The subgraph with percolating clusters is a percolating subgraph and is denoted by G'_p , otherwise the subgraph is a non-percolating subgraph. Then, we have

$$E_p(L, p) = \sum_{G'_p} p^{b(G'_p)} (1-p)^{E-b(G'_p)} \quad (1)$$

where $b(G'_p)$ is the number of occupied bonds in G'_p .

To carry out HMCSM, we first choose w different values of p . For a given $p = p_j$, $1 \leq j \leq w$, we generate N_R different subgraphs G' . The data obtained from the wN_R different G' are then used to construct two arrays of numbers of length E with elements $N_p(b)$ and $N_f(b)$, $0 \leq b \leq E$, which are, respectively, the total numbers of percolating subgraphs with b occupied bonds and non-percolating subgraphs with b occupied bonds. If wN_R is very large, we can use the histograms to calculate approximate E_p for any value of the bond occupation probability p [2]:

$$E_p(L, p) = \sum_{b=0}^E p^b (1-p)^{E-b} C_b^E \frac{N_p(b)}{N_p(b) + N_f(b)}. \quad (2)$$

If we obtain $E_p(L_1, p)$ and $E_p(L_2, p)$ for two similar lattices G_1 and G_2 of linear

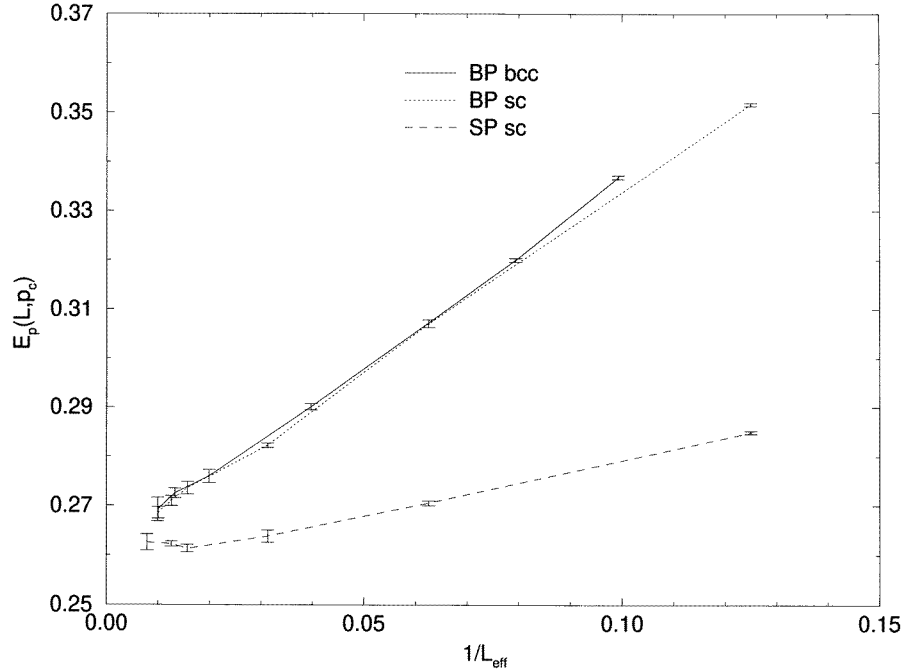


Figure 3. The calculated critical $E_p(L, p_c)$ for BP on bcc lattices, BP on sc lattices, and SP on sc lattices as a function of $1/L_{\text{eff}}$.

dimensions L_1 and L_2 , respectively, the RG transformation from G_1 to G_2 is given by

$$E_p(L_2, p') = E_p(L_1, p) \quad (3)$$

which gives the renormalized p' as a function of p . The fixed point of (3) gives the critical point p_c , i.e.

$$E_p(L_2, p_c) = E_p(L_1, p_c). \quad (4)$$

The p_c of (4) depends on L_1 and L_2 and will be denoted by $p_c(L_1, L_2)$. For site percolation on square lattices, Hu *et al* [18] found that $p_c(L_1, L_2)$ obtained by cell-to-cell RG transformation approaches its limiting value, p_c , quicker than cell-to-site RG transformation and the cell-to-cell scheme is insensitive to the scaling power used in the extrapolation of p_c .

We use (2) to evaluate $E_p(L, p)$ for bond percolation (BP) on sc and bcc lattices and a similar equation to evaluate $E_p(L, p)$ for site percolation (SP) on sc lattices for several linear dimensions of the lattices. Typical results for BP on $60 \times 60 \times 60$ and $80 \times 80 \times 80$ bcc lattices, BP on $80 \times 80 \times 80$ and $100 \times 100 \times 100$ sc lattices, and SP on $80 \times 80 \times 80$ and $128 \times 128 \times 128$ sc lattices are shown in figure 1.

For BP on sc lattices, pairs of linear dimensions (L_1, L_2) for equation (4) are chosen to be (16, 8), (32, 16), (64, 32), (80, 64), and (100, 80) and the corresponding $p_c(L_1, L_2)$ are 0.25241 ± 0.00012 , 0.24976 ± 0.00009 , 0.24897 ± 0.00005 , 0.24890 ± 0.00006 , and 0.24887 ± 0.00006 , respectively. We plot these data as a function of $(L_1)_{\text{eff}}^{-1/\nu}$ in figure 2, where the numerical value of $\nu (= \nu^{-1}) = 1.14 \pm 0.01$ [19] is used. As $(L_1)_{\text{eff}}^{-1/\nu}$ approaches zero, the values of $p_c(L_1, L_2)$ approach the value of 0.248810 ± 0.000050 obtained by Ziff and Stell [19], which is represented by a down triangle on the vertical axis of figure 2.

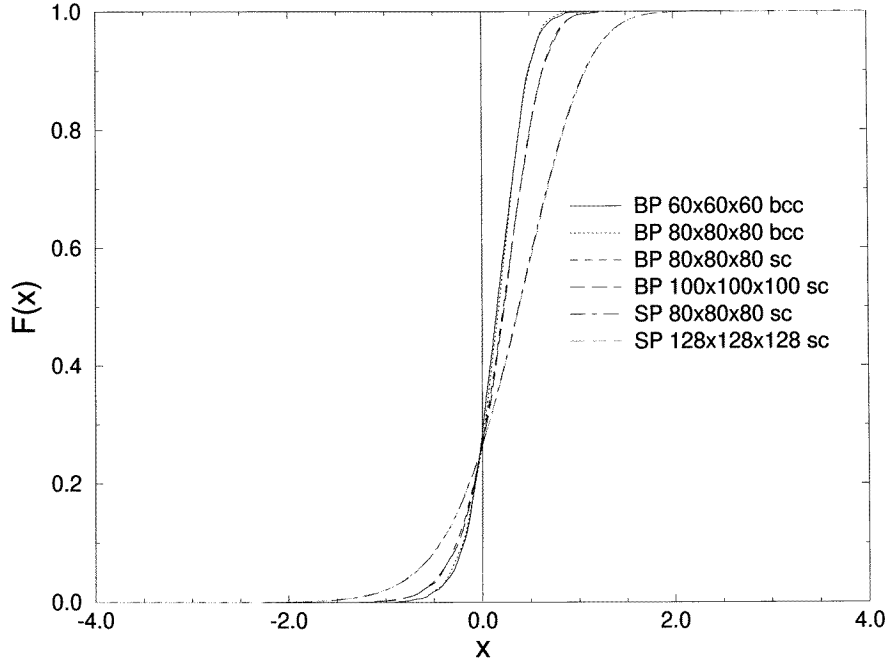


Figure 4. The calculated E_p of BP on bcc lattices, BP on sc lattices, and SP on sc lattices as a function of x , where $x = (p - p_c)L_{\text{eff}}^{y_t}$. The finite-size scaling function is $F(x)$.

For SP on sc lattices, pairs of linear dimensions (L_1, L_2) are chosen to be (16, 8), (32, 16), (64, 32), (80, 64), and (128, 80) and the corresponding $p_c(L_1, L_2)$ are 0.31377 ± 0.00008 , 0.312065 ± 0.00008 , 0.31170 ± 0.00009 , 0.311537 ± 0.00011 , and 0.31162 ± 0.00012 , respectively. These values shown in figure 2 approach the value of 0.311605 ± 0.000010 obtained by Ziff and Stell [19], which is denoted by a square on the vertical axis of figure 2. For BP on bcc lattices, pairs of linear dimensions (L_1, L_2) are chosen to be (10, 8), (20, 10), (40, 20), (60, 40), and (80, 60) and the corresponding $p_c(L_1, L_2)$ are 0.18367 ± 0.00012 , 0.18144 ± 0.00006 , 0.18055 ± 0.00007 , 0.18036 ± 0.00007 , and 0.18032 ± 0.00006 , respectively. These values shown in figure 2 approach the value of 0.18025 ± 0.00015 obtained by Adler *et al* [20], which is denoted by a up triangle on the vertical axis of figure 2.

The critical $E_p(L, p_c)$'s evaluated at $p_c = 0.2488$, 0.3116 , and 0.1803 for BP on sc lattices, SP on sc lattices, and BP on bcc lattices, respectively, are shown in figure 3, in which the horizontal axis is $1/L_{\text{eff}}$. The numbers of samples (i.e. the value of N_R) for BP on sc lattices are 50 000, 90 000, 90 000, 90 000, 450 000, and 450 000 for $L = 100, 80, 64, 32, 16,$ and 8 , respectively. The numbers of samples for BP on bcc lattices are 50 000, 90 000, 90 000, 270 000, 450 000, and 900 000 for $L = 80, 60, 40, 20, 10,$ and 8 , respectively. The numbers of samples for SP on sc lattices are 90 000, 90 000, 180 000, 180 000, 450 000, and 900 000 for $L = 128, 64, 32, 16,$ and 8 , respectively. Based on visual evaluation of figure 3, we extrapolate that the curves for BP and SP on sc lattices and BP on bcc lattices all approach values of 0.265 ± 0.005 as $1/L_{\text{eff}} \rightarrow 0$.

Using the numerical values of $y_t (= \nu^{-1}) = 1.14 \pm 0.01$ [19] and p_c calculated in this letter, we plot $E_p(L, p)$ as a function of $x = (p - p_c)L_{\text{eff}}^{y_t}$ in figure 4, which shows that

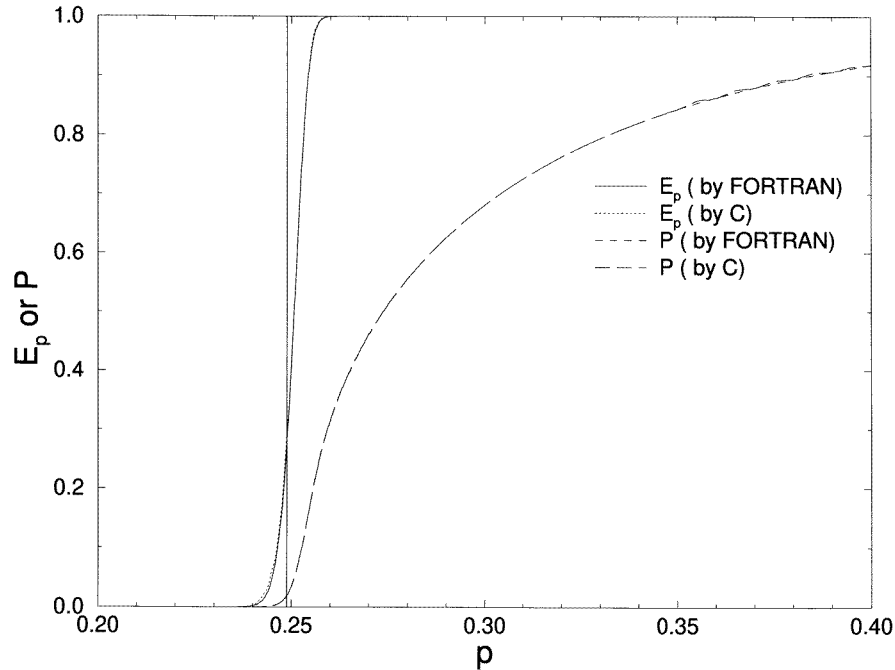


Figure 5. Results for two independent calculations of E_p and the percolation probability P as a function of p for BP on a $64 \times 64 \times 64$ sc lattice with free boundary conditions and R_1 spanning rule. The E_p functions are represented by full and dotted curves. The P functions are represented by dashed and long-dashed curves. The vertical full line goes through the critical point p_c .

E_p has well defined finite-size scaling behaviour for these three models and the obtained finite-size scaling function is denoted by $F(x)$. Since we obtain the nice finite-size scaling behaviour shown in figure 4, we expect that our critical value $E_p = 0.265 \pm 0.005$ can well represent $E_p(L, p_c)$ as $L \rightarrow \infty$. However, our value $E_p = 0.265 \pm 0.005$ is quite different from 0.42 obtained by Stauffer *et al* [16].

To check the reliability of our result, we used both FORTRAN and C computer languages and different random number generators to prepare two independent computer programs. Typical calculated results for E_p and the percolation probability P of bond percolation on a $64 \times 64 \times 64$ sc lattice are plotted in figure 5, which shows that the results obtained from two programs are consistent.

In [21], Hu pointed out that Hu, Lin and Chen (HLC) [11] and Hovi and Aharony (HA) [22] used different definitions of periodic boundary conditions so that HLC and HA obtained different critical E_p for $L \times L$ square lattices. The inconsistency of our critical E_p , 0.265 ± 0.005 , and that of Stauffer *et al* [16], 0.42, suggests that the two groups use different definitions of *free boundary conditions*.

In summary, we have found universality of critical E_p for site and bond percolation on sc and bcc lattices. We have also found universality of critical W_n for site and bond percolation on 3d lattices, which will be published elsewhere [23].

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References

- [1] Reynolds P J, Stanley H E and Klein W 1980 *Phys. Rev. B* **21** 1223
- [2] Hu C-K 1992 *Phys. Rev. B* **46** 6592
- [3] Hu C-K 1992 *Phys. Rev. Lett.* **69** 2739
- [4] Langlands R P, Pichet C, Pouliot Ph and Saint-Aubin Y 1992 *J. Stat. Phys.* **67** 553
- [5] Cardy J L 1992 *J. Phys. A: Math. Gen.* **25** L201
- [6] Ziff R M 1992 *Phys. Rev. Lett.* **69** 2670
- [7] Stauffer D and Aharony A 1992 *Introduction to Percolation Theory* 2nd edn (London: Taylor and Francis)
- [8] Hu C-K 1994 *J. Phys. A: Math. Gen.* **27** L813
- [9] Hu C-K 1994 *Chin. J. Phys. (Taipei)* **32** 519
- [10] Hu C-K and Chen J-A 1995 *J. Phys. A: Math. Gen.* **28** L73
- [11] Hu C-K, Lin C-Y and Chen J-A 1995 *Phys. Rev. Lett.* **75** 193
Hu C-K, Lin C-Y and Chen J-A 1995 *Phys. Rev. Lett.* **75** 2786(E)
- [12] Hu C-K, Lin C-Y and Chen J-A 1995 *Physica A* **221** 80
- [13] Hu C-K 1986 *J. Korean Phys. Soc. (Proc. Suppl.)* **29** S97–101
- [14] Hu C-K and Lin C-Y 1996 *Phys. Rev. Lett.* **77** 8
- [15] Privman V and Fisher M E 1984 *Phys. Rev. B* **30** 322
- [16] Stauffer D, Adler J and Aharony A 1994 *J. Phys. A: Math. Gen.* **27** L475
- [17] Gropengiesser U and Stauffer D 1994 *Physica A* **210** 320
- [18] Hu C-K, Chen C-N and Wu F Y 1996 *J. Stat. Phys.* **82** 1199
- [19] Ziff R M and Stell G 1988 Report No 88–4, University of Michigan
- [20] Adler J, Meir Y, Aharony A and Harris A B 1990 *Phys. Rev. B* **41** 9183 and references therein.
- [21] Hu C-K 1996 *Phys. Rev. Lett.* **76** 3875
- [22] Hovi J-P and Aharony A 1996 *Phys. Rev. Lett.* **76** 3874
- [23] Lin C-Y and Hu C-K 1998 *Preprint* to be published